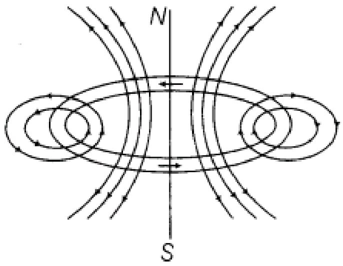
Magnetic Field Laws & their Applications

1 Mark Question

1.Draw the magnetic field lines due to a current carrying loop. [Delhi 2013 C]
Ans.

Magnetic field lines due to a current carrying loop are given by



(1)

2 Marks Questions

2.Considering the case of a parallel plate capacitor being charged, show how one is required to generalise Ampere's circuital law to include the term due to displacement current. [All India 2014,2011]
Ans.

Ampere's circuital law conduction current during charging of a capacitor was found inconsistent. Therefore, Maxwell modified Ampere's circuital law by introducing displacement current. **(1/2)**

Ampere's circuital law $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$ was modified to $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 (I_R + I_D)$

It is called modified Ampere's circuital law or Ampere-Maxwell's circuital law.

The displacement current arising due to time varying electric field is given by $I_D = \varepsilon_0 \frac{d\phi_E}{dt}$

Therefore, modified Ampere's circuital law may be expressed as

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(I_C + \varepsilon_0 \frac{d\phi_E}{dt} \right)$$ **(1/2)**

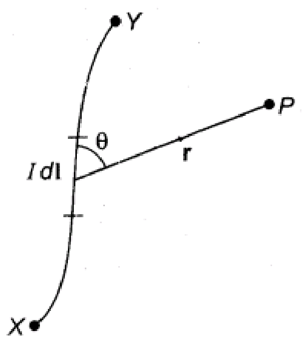
3.(i) State Biot-Savart's law in vector form expressing the magnetic field due to an element dl carrying current 7 at a distance r from the element.
(ii) Write the expression for the magnitude of the magnetic field at the centre of a circular loop of radius r carrying a steady current 7. Draw the field lines due to the current loop.[All India 2014C]

Ans.(i)

Biot-Savart's law This law states that the magnetic field (dB) at point P due to small current element Idl of current carrying conductor is

(*i*) directly proportional to the Idl (current) element of the conductor.

$dB \propto Idl$



(*ii*) directly proportional to $\sin\theta$

$dB \propto \sin\theta$

where, θ is the angle between dl and **r**.

(*iii*) inversely proportional to the square of the distance of point P from the current element.

$$dB \propto \frac{1}{r^2}$$ **(1)**

Combining all the inequalities

$$dB \propto \frac{Idl\sin\theta}{r^2}$$

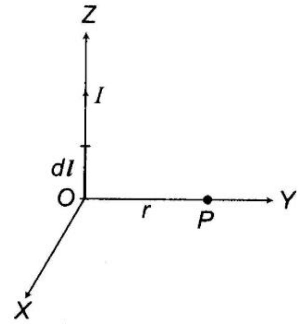
$$dB = \frac{\mu_0}{4\pi}\cdot\frac{Idl\sin\theta}{r^2}$$

where, $\frac{\mu_0}{4\pi} = 10^{-7}$ T-m/A for free space.

The direction of magnetic field can be obtained using right hand thumb rule.

In vector form,

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\cdot\frac{Id\mathbf{l}\times\hat{\mathbf{r}}}{|\mathbf{r}|^2}$$ **(1/2)**

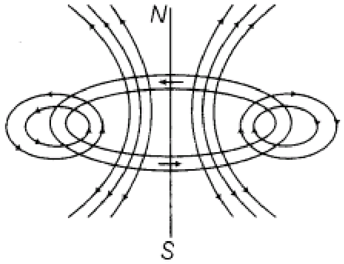


(1/2)

The direction of magnetic field will be perpendicular to Y-axis along upward in the plane of paper.

(ii)

Magnetic field lines due to a current carrying loop are given by



(1)

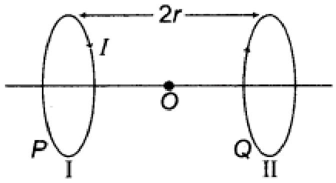
4.Define one tesla using the expression for the magnetic force acting on a particle of charge q moving with velocity v in a magnetic field B. [Foreign 2014]

Ans.

One tesla is defined as the field which produces a force of 1 newton when a charge of 1 coulomb moves perpendicularly in the region of the magnetic field at a velocity of 1m/s.

$$F = qvB \Rightarrow B = \frac{f}{qv} \Rightarrow 1\text{T} = \frac{1\text{ N}}{(1\text{C})(1\text{ m/s})} \quad \textbf{(2)}$$

5.Two identical circular loops P and Q, each of radius r and carrying equal currents are kept in the parallel planes having a common axis passing through O. The direction of current in P is clockwise and in Q is anti-clockwise as seen from O which is equidistant from the loops P and Q. Find the magnitude of the net magnetic field at O.



[Delhi 2012]

Ans.

> To calculate net magnetic field at point O, first of all, calculate the magnetic field at point O due to both coils with direction. By vector addition of these two magnetic fields, net magnetic field can be obtained.

Magnetic field at O due to two rings will be in same direction ($Q \to P$, along the axis) and of equal magnitude. **(1/2)**

$$B = B_1 + B_2 \text{ but } B_2 = B_1$$

$$\Rightarrow \quad B = 2B_1 = 2\left[\frac{\mu_0 I r^2}{2(r^2 + r^2)^{3/2}}\right] \quad \textbf{(1/2)}$$

$$B = \frac{\mu_0 I r^2}{(2r^2)^{3/2}} = \frac{\mu_0 I r^2}{2^{3/2} r^3} \quad \textbf{(1/2)}$$

$$B = \frac{\mu_0 I}{2^{3/2} r} \quad \textbf{(1/2)}$$

6.A circular coil of closely wound N turns and radius r carries a current 7. Write the expressions for the following:

(i)The magnetic field at its centre.

(ii)The magnetic moment of this COil. [All India 2012]

Ans.

(*i*) Magnetic field at centre due to circular current carrying coil, $B = \frac{\mu_0 NI}{2r}$ **(1)**

(*ii*) Magnetic moment, $M = NIA = NI(\pi r^2)$

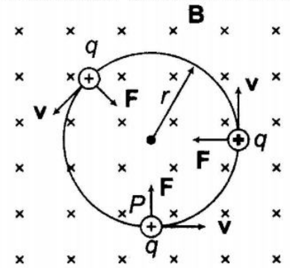
$$M = \pi NIr^2 \quad \textbf{(1)}$$

where, r is the radius of circular coil, μ_0 is permeability of free space and N is number of turns.

7.A particle of charge q and mass m is moving with velocity It is subjected to a uniform magnetic field B directed perpendicular to its velocity. Show that it describes a circular path. Write the expression for its radius.[Foreign 2012]

Ans.

A charge q projected perpendicular to the uniform magnetic field B with velocity v. The perpendicular force, $F = qvB$, acts like a centripetal force perpendicular to the magnetic field. Then, the path followed by charge is circular as shown in the figure. **(1)**



The Lorentz magnetic force acts as a centripetal force, thus

$$qvB = \frac{mv^2}{r} \quad \text{or} \quad r = \frac{mv}{qB}$$

where, r = radius of the circular path followed by charge projected perpendicular to a uniform magnetic field. **(1)**

8. Show how the equation for Ampere's circuital law, *viz*

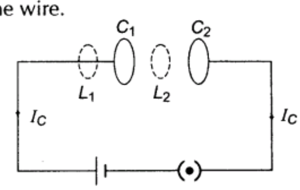
$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

is modified in the presence of displacement current.

[Foreign 2011]

Ans.

During charging of a capacitor, let at any instant transient current (I_C) flows through the wire.



Applying Ampere,s circuital law for loops L_1 and L_2, we get

$$\oint_{L_1} \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_C$$

$$\oint_{L_2} \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times 0 = 0$$

This violates the concept of continuity of electric current. **(1)**

Maxwell introduced the concept of displacement current flowing in space due to varying electric field such that

$$I_C = I_D = \varepsilon_0 \frac{d\phi_E}{dt}$$

This maintained continuity of current.

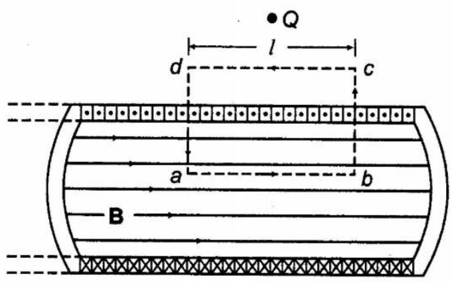
∴ Modified Ampere's circuital law is given by

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 (I_C + I_D).$$ **(1)**

9.A long solenoid of length L having N turns carries a current I.Deduce the expression for the magnetic field in the interior of the solenoid. [All India 2008; 2011C]

Ans.

Figure shows the longitudinal sectional view of long current carrying solenoid. The current comes out of the plane of paper at points marked.



The **B** is the magnetic field at any point inside the solenoid.

Considering the rectangular closed path *abcda*. Applying Ampere's circuital law over loop *abcda*. **(1)**

$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times$ (Total current passing through loop *abcd*)

$$\int_a^b \mathbf{B} \cdot d\mathbf{l} + \int_b^c \mathbf{B} \cdot d\mathbf{l} + \int_c^d \mathbf{B} \cdot d\mathbf{l} + \int_d^a \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(\frac{N}{L} li\right)$$

where, $\frac{N}{L}$ = number of turns per unit length

$ab = cd = l$ = length of rectangle.

$$\int_a^b Bdl \cos 0° + \int_b^c Bdl \cos 90° + 0 + \int_d^a Bdl \cos 90° = \mu_0 \left(\frac{N}{L}\right) li$$

$$B \int_a^b dl = \mu_0 \left(\frac{N}{L}\right) li \Rightarrow Bl = \mu_0 \left(\frac{N}{L}\right) li$$

$$\Rightarrow \quad B = \mu_0 \left(\frac{N}{L}\right) i \quad \text{or} \quad B = \mu_0 ni$$ **(1)**

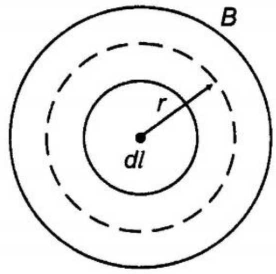
where number of turns per unit length. This is required expression for magnetic field inside the long current carrying solenoid.

10.Obtain with the help of a necessary diagram, the expression for the magnetic field in the interior of a toroid carrying current. [HOTS; All India 2011C]

Ans.

• Toroid is an endless solenoid to calculate the magnetic field in the interior of toroid, Ampere's circuital law can be obtained.

Toroid is a hollow circular ring on which a large number of insulated turns of a metallic wire are closely wound. The direction of the magnetic field at a point is given by tangent to the magnetic field line at that point. **(1)**



$\oint \mathbf{B}\, dl = \int B dl \cos 0° = B\, 2\pi r$ ....(i)

as $\oint \mathbf{B} \cdot dl = \mu_0 I \times$ Number of turns ....(ii)

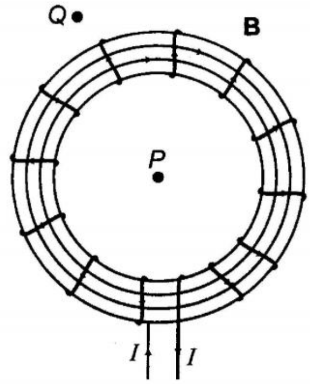
If n be the number of turns/unit length, then total number of turns $= n \times 2\pi r$

so $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 n \times 2\pi r I$ ....(iii)

From Eqs. (i) and (iii), we get

$$B\, 2\pi r = \mu_0 n 2\pi r I$$

$$B = \mu_0 n I$$



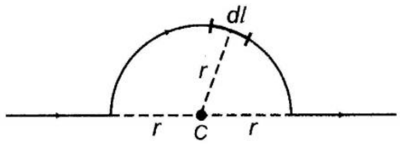
Applying Ampere's circuital law over loop, we have **(1)**

$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times$ Current passing through the loop

11.A straight wire of length L is bent into a semi-circular loop. Use Biot-Savart's law to deduce an expression for the magnetic field at its centre due to the current 7 passing through it. [Delhi 2011c]

Ans.

. When a straight wire is bent into semi-circular loop, then there are two parts which can produce the magnetic field at the centre one is circular part and other is straight part due to which field is zero.



∵ Length L is bent into semi-circular loop.

Length of wire = Circumference of semi-equal circular wire

$$\Rightarrow \quad L = \pi r$$

$$r = \frac{L}{\pi} \quad \text{...(i)}$$

Considering a small element dl on current loop. The magnetic field dB due to small current element Idl at centre C.

Using Biot-Savart's law, we have

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin 90°}{r^2} \quad \textbf{(1/2)}$$

$$[\because Idl \perp \mathbf{r}, \therefore \theta = 90°]$$

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl}{r^2}$$

∴ Net magnetic field at C due to semi-circular loop,

$$B = \int_{\text{semicircle}} \frac{\mu_0}{4\pi} \frac{Idl}{r^2}$$

$$B = \frac{\mu_0}{4\pi} \frac{I}{r^2} \int_{\text{semicircle}} dl \quad \textbf{(1/2)}$$

$$B = \frac{\mu_0}{4\pi} \cdot \frac{I}{r^2} L$$

But, $r = \frac{L}{\pi}$

$$B = \frac{\mu_0}{4\pi} \cdot \frac{IL}{(L/\pi)^2} = \frac{\mu_0}{4\pi} \times \frac{IL}{L^2} \times \pi^2$$

$$\Rightarrow \quad B = \frac{\mu_0 I\pi}{4L} \quad \textbf{(1)}$$

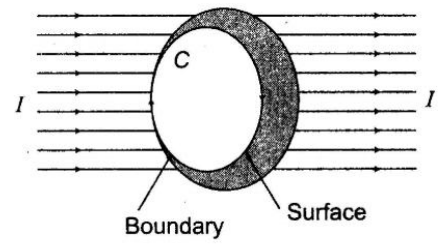
This is required expression.

12.State Ampere's circuital law. Show through an example, how this law enables an easy evaluation of the magnetic field when there is a symmetry in the system? [All India 2010]

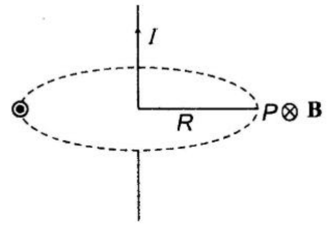
Ans.

As, Ampere's circuital law states that the line integral of magnetic field **B** around any closed loop is equal to μ_0 times the total current threading through the loop. **(1)**

i.e. $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$



To explain the Ampere's circuital law consider an infinitely long conductor wire carrying a steady current I as shown in the figure.



In order to determine the magnetic field at point P which is situated at a distance R from the centre of the circular loop around the conductor wire **B** (magnetic field) is tangential to circumference of the loop. **(1)**

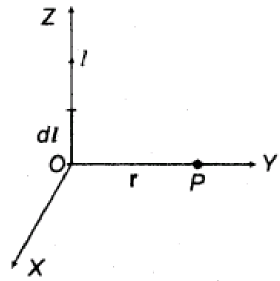
Now, $\oint \mathbf{B} \cdot d\mathbf{l} = \int B\,dl = B\,2\pi R$

$$= \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi R}$$

[From Ampere's circuital law]

The direction of magnetic field will be determined by right hand rule.

13.State Biot-Savart's law. A current I flows in a conductor placed perpendicular to the plane of the paper. Indicate the direction of the magnetic field due to a small element dl at a point P situated at a distance r from the element as shown in the figure.[Delhi 2009]



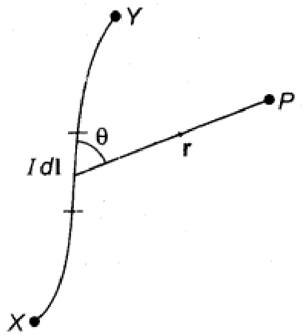
[Delhi 2009]

Ans.

Biot-Savart's law This law states that the magnetic field (dB) at point P due to small current element Idl of current carrying conductor is

(*i*) directly proportional to the Idl (current) element of the conductor.

$$dB \propto Idl$$



(*ii*) directly proportional to $\sin\theta$

$$dB \propto \sin\theta$$

where, θ is the angle between dl and **r**.

(*iii*) inversely proportional to the square of the distance of point P from the current element.

$$dB \propto \frac{1}{r^2}$$ **(1)**

Combining all the inequalities

$$dB \propto \frac{Idl \sin\theta}{r^2}$$

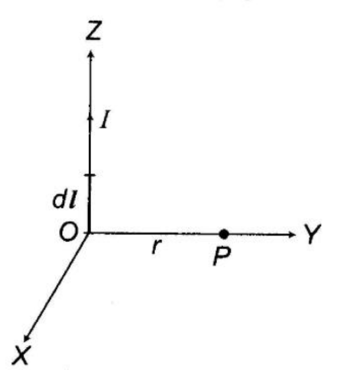
$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin\theta}{r^2}$$

where, $\frac{\mu_0}{4\pi} = 10^{-7}$ T-m/A for free space.

The direction of magnetic field can be obtained using right hand thumb rule.

In vector form,

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{Id\mathbf{l} \times \hat{\mathbf{r}}}{|\mathbf{r}|^2}$$ **(1/2)**



(1/2)

The direction of magnetic field will be perpendicular to Y-axis along upward in the plane of paper.

14.A wire of length L is bent round in the form of a coil having N turns of same radius. If a steady current I flows through it in clockwise direction, then find the magnitude and direction of the magnetic field produced at its centre.[Foreign 2009]

Ans.

> When a straight wire is bent in the form of a circular coil of N turns, then the length of the wire is equal to circumference of the coil multiplied by the number of turns.

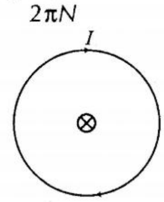
Let the radius of coil be r.

As, the wire is bent round in the form of a coil having N turns.

$\therefore$ $N \times$ circumference of the coil = Length of the wire

$\therefore \quad (2\pi r) \times N = L$

$$r = \frac{L}{2\pi N} \qquad \text{...(i)}$$



Magnetic field at the centre due to N turns of a coil is given by

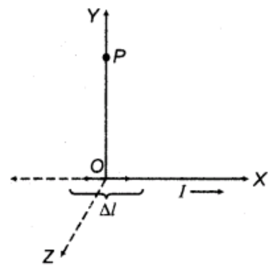
$$B = \frac{\mu_0 (NI)}{2r}$$

$$B = \frac{\mu_0 (NI)}{2\left(\frac{L}{2\pi N}\right)} \qquad \text{[From Eq. (i)]}$$

$$B = \frac{\mu_0 \pi N^2 I}{L}$$ **(1½)**

The direction of magnetic field is perpendicular to the plane of loop and entering into it. **(1/2)**

15. An element $\Delta l = \Delta x I$ is placed at the origin (as shown in figure) and carries a current $I = 2$ A. Find out the magnetic field at a point P on the Y-axis at a distance of 1.0 m due to the element $\Delta x = w$ cm. Also, give the direction of the field produced. **[Delhi 2009C]**



Ans.

Biot-Savart's law states that

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{Id\mathbf{l} \times \hat{\mathbf{r}}}{|\mathbf{r}|^2}$$ **(1)**

Here, $Idl = 2 \times w\hat{\mathbf{i}}$

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{(2w\hat{\mathbf{i}} \times \hat{\mathbf{j}})}{(1)^2}$$ **(1/2)**

$\hat{\mathbf{r}} = \hat{\mathbf{j}}$

$|\mathbf{r}| = 1\text{m}$

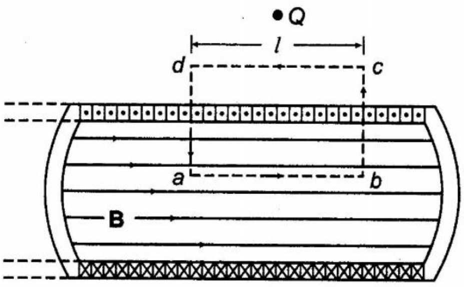
$$d\mathbf{B} = \frac{\mu_0 w}{2\pi}\hat{\mathbf{k}} \Rightarrow |d\mathbf{B}| = \frac{\mu_0 w}{2\pi}$$

and direction along $+Z$-axis. **(1/2)**

16.Using Ampere's circuital law,obtain an expression for the magnetic field along the axis of a current carrying solenoid of length / and having N number of turns. [All India 2008]

Ans.

Figure shows the longitudinal sectional view of long current carrying solenoid. The current comes out of the plane of paper at points marked.



The **B** is the magnetic field at any point inside the solenoid.

Considering the rectangular closed path *abcda*. Applying Ampere's circuital law over loop *abcda*. **(1)**

$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times$ (Total current passing through loop *abcd*)

$$\int_a^b \mathbf{B} \cdot d\mathbf{l} + \int_b^c \mathbf{B} \cdot d\mathbf{l} + \int_c^d \mathbf{B} \cdot d\mathbf{l} + \int_d^a \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(\frac{N}{L} li\right)$$

where, $\frac{N}{L}$ = number of turns per unit length

$ab = cd = l$ = length of rectangle.

$$\int_a^b Bdl \cos 0° + \int_b^c Bdl \cos 90° + 0 + \int_d^a Bdl \cos 90° = \mu_0 \left(\frac{N}{L}\right) li$$

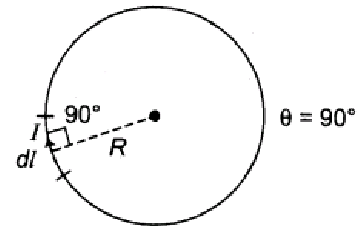
$$B\int_a^b dl = \mu_0 \left(\frac{N}{L}\right) li \Rightarrow Bl = \mu_0 \left(\frac{N}{L}\right) li$$

$$\Rightarrow \quad B = \mu_0 \left(\frac{N}{L}\right) i \quad \text{or} \quad B = \mu_0 ni$$ **(1)**

17.A circular coil of radius R carries a current /. Write the expression for the magnetic field due to this coil at its centre. Find out the direction Of the field. [All India 2008 C]

Ans.

Consider a small element dl on a circular coil of radius R carrying current I.



∴ By Biot-Savart's law, magnetic field at the centre due to element of coil.

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin 90°}{R^2}$$ **(1/2)**

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{I}{R^2} dl$$

∴ Net magnetic field at centre,

$$B = \oint \frac{\mu_0}{4\pi} \frac{I}{R^2} dl$$

$$= \frac{\mu_0}{4\pi} \frac{I}{R^2} \oint dl$$

$$= \frac{\mu_0}{4\pi} \frac{I}{R^2} \times 2\pi R$$

$$= \frac{\mu_0 I}{2R}$$

For N turns of coil, $B = \frac{\mu_0 NI}{2R}$ **(1)**

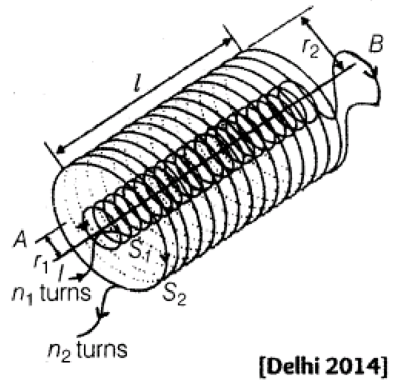
The direction of magnetic field is perpendicular to the plane of paper and directed inwards to the plane. **(1/2)**

3 Marks Questions

18.(i) State Ampere's circuital law expressing it in the integral form, (ii) Two long co-axial insulated solenoids and S_2 of equal length are wound one over the other as shown in the figure.A steady current / flows through the inner solenoid S_x to the other end B which is connected to the outer solenoid through which the some current / flows in the opposite direction so, as to come out at end A. If n_x and n_2 are the number of turns per unit length, find the magnitude and direction of the net magnetic field at a point

(a)inside on the axis and

(b)outside the combined system



[Delhi 2014]

Ans.

(*i*) Ampere's circuital law states that the line integral of magnetic field (*B*) around any closed path in vacuum is μ_0 times the net current (*I*) threading the area enclosed by the curve.

Mathematically, $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$

Ampere's law is applicable only for an Amperian loop as the Gauss's law is used for gaussian surface in electrostatics. **(1)**

(*ii*) According to Ampere's circuital law, the net magnetic field is given by $B = \mu_0 n \hat{\mathbf{i}}$ **(1)**

(a) The net magnetic field is given by

$B_{net} = B_2 - B_1$

$= \mu_0 n_2 I_2 - \mu_0 n_1 I_1 \quad [\because I_2 = I_1 = I]$

$= \mu_0 I (n_2 - n_1)$

The direction is from *B* to *A*. **(1)**

(*b*) As the magnetic field due to S_1 is confined solely inside S_1 as the solenoids are assumed to be very long. So, there is no magnetic field outside S_1 due to current in S_1, similarly there is no field outside S_2.

$\therefore \quad B_{net} = 0$ **(1)**

19.(i) How is a toroid different from a solenoid?

(ii) Use Ampere's circuital law to obtain the magnetic field inside a toroid.

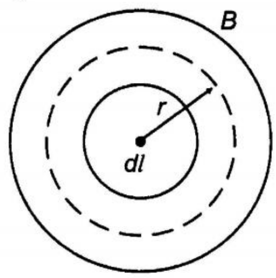
(iii)Show that in an ideal toroid the [1] magnetic field (a) inside the toroid and (b) outside the toroid at any point in the open space is zero. [All India 2014 C]

Ans.(i) A toroid can be viewed as a solenoid which has been bent into circular shape to close on itself.

(ii)

- Toroid is an endless solenoid to calculate the magnetic field in the interior of toroid, Ampere's circuital law can be obtained.

Toroid is a hollow circular ring on which a large number of insulated turns of a metallic wire are closely wound. The direction of the magnetic field at a point is given by tangent to the magnetic field line at that point. **(1)**



$\oint \mathbf{B}\, dl = \int B dl \cos 0° = B\, 2\pi r$ ....(i)

as $\oint \mathbf{B} \cdot dl = \mu_0 I \times$ Number of turns ....(ii)

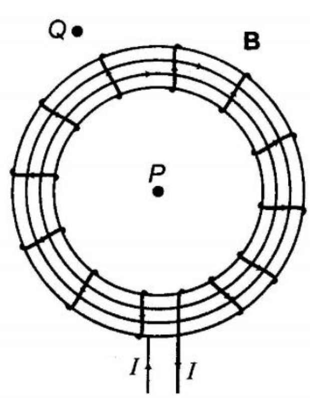
If n be the number of turns/unit length, then total number of turns $= n \times 2\pi r$

so $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 n \times 2\pi r I$ ....(iii)

From Eqs. (i) and (iii), we get

$$B\, 2\pi r = \mu_0 n 2\pi r I$$

$$B = \mu_0 n I$$

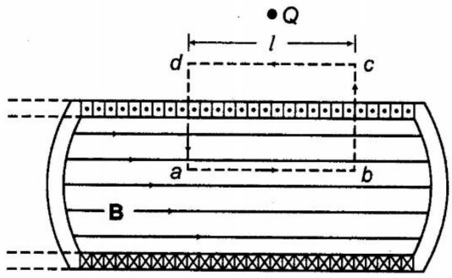


Applying Ampere's circuital law over loop, we have **(1)**

$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times$ Current passing through the loop

(iii)For the evaluation of magnetic field for a symmetrical system, we can consider the example of a current carrying solenoid. Now,

Figure shows the longitudinal sectional view of long current carrying solenoid. The current comes out of the plane of paper at points marked.



The **B** is the magnetic field at any point inside the solenoid.

Considering the rectangular closed path *abcda*. Applying Ampere's circuital law over loop *abcda*. **(1)**

$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times$ (Total current passing through loop *abcd*)

$$\int_a^b \mathbf{B} \cdot d\mathbf{l} + \int_b^c \mathbf{B} \cdot d\mathbf{l} + \int_c^d \mathbf{B} \cdot d\mathbf{l} + \int_d^a \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(\frac{N}{L} li\right)$$

where, $\frac{N}{L}$ = number of turns per unit length

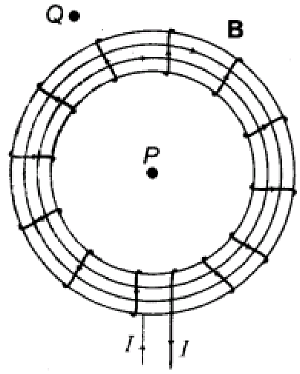
$ab = cd = l$ = length of rectangle.

$$\int_a^b Bdl \cos 0° + \int_b^c Bdl \cos 90° + 0 + \int_d^a Bdl \cos 90° = \mu_0 \left(\frac{N}{L}\right) li$$

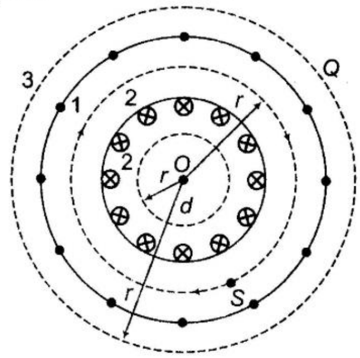
$$B \int_a^b dl = \mu_0 \left(\frac{N}{L}\right) li \Rightarrow Bl = \mu_0 \left(\frac{N}{L}\right) li$$

$$\Rightarrow \qquad B = \mu_0 \left(\frac{N}{L}\right) i \quad \text{or} \quad B = \mu_0 ni$$ **(1)**

where number of turns per unit length. This is required expression for magnetic field inside the long current carrying solenoid.



(a) Let magnetic field inside the toroid is B along the considered loop (1) as shown in figure.



Applying Ampere's circuital law,

$$\oint_{loop\,1} \mathbf{B} \cdot d\mathbf{l} = \mu_0 (NI)$$

Since, toroid of N turns, threads the loop 1, N times, each carrying current I inside the loop. Therefore, total current threading the loop 1 is NI.

$$\Rightarrow \oint_{loop\,1} \mathbf{B} \cdot d\mathbf{l} = \mu_0 NI$$

$$B \oint_{loop} dl = \mu_0 NI$$

$$B \times 2\pi r = \mu_0 NI \quad \text{or} \quad B = \frac{\mu_0 NI}{2\pi r}$$ **(1)**

(b) **Magnetic field inside the open space interior the toroid.** Let the loop (2) is shown in figure experience magnetic field B.

No current threads the loop 2 which lie in the open space inside the toroid.

∴ Ampere's circuital law

$$\oint_{loop\,2} \mathbf{B} \cdot d\mathbf{l} = \mu_0 (\theta) = 0 \Rightarrow B = 0$$ **(1)**

Magnetic field in the open space exterior of toroid Let us consider a coplanar loop (3) in the open space of exterior of toroid. Here, each turn of toroid threads the loop two times in opposite directions.

Therefore, net current threading the loop

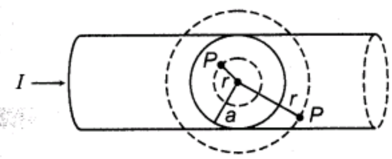
$$= NI - NI = 0$$

∴ By Ampere's circuital law,

$$\oint_{loop\,3} \mathbf{B} \cdot d\mathbf{l} = \mu_0 (NI - NI) = 0 \Rightarrow B = 0$$

Thus, there is no magnetic field in the open space interior and exterior of toroid. **(1)**

20. Figure shows a long straight wire of a circular cross-section of radius a carrying steady current I. The current I is uniformly distributed across this cross-section. Derive the expressions for the magnetic field in the region $r < a$ and $r > a$.

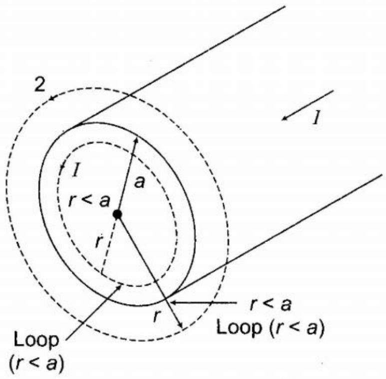


[All India 2011C]

Ans.

In these types of questions, first of all we have to calculate the current per unit area of cross-section so that we can calculate the current in each loop, then only we can find the magnetic field.

The current is distributed uniformly across the cross-section of radius a.



$\therefore$ Current passes per unit cross-section $= \dfrac{I}{\pi a^2}$

$\therefore$ Current passes through the cross-section of radius r is

$$I' = \left(\frac{I}{\pi a^2} \times \pi r^2\right) = \frac{Ir^2}{a^2} \quad \text{...(i)} \ \textbf{(1/2)}$$

(i) Consider a loop of radius r whose centre lies at the axis of wire where, $r < a$ as shown in figure inside the wire.

Applying Ampere's circuital law,

$$\oint \mathbf{B} \cdot dl = \mu_0 I' \quad \textbf{(1/2)}$$

$$\oint Bdl \cos 0^\circ = \mu_0 \left(\frac{Ir^2}{a^2}\right) \quad \text{[From Eq. (i)]}$$

$$B \oint dl = \mu_0 \frac{Ir^2}{a^2}$$

$$B \times 2\pi r = \frac{\mu_0 Ir^2}{a^2} \Rightarrow B = \frac{\mu_0 Ir}{2\pi a^2} \quad \textbf{(1/2)}$$

$$\Rightarrow \quad B \propto r$$

(*ii*) Considering a loop of radius r whose centre lies at the axis of wire and ($r > a$) as shown in outer dotted line.

∴ Current I threads the loops.

Applying Ampere's circuital law,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$ **(1/2)**

$$\oint Bdl \cos 0° = \mu_0 I$$

$$B \oint dl = \mu_0 I$$

$$B \times 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$ **(1)**

$$\Rightarrow \quad B \propto \frac{1}{r}$$

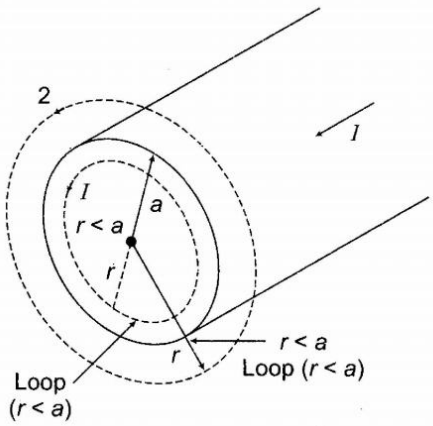
21. A long straight wire of a circular cross-section of radius a carries a steady current I. The current is uniformly distributed across the cross-section. Apply Ampere's circuital law to calculate the magnetic field at a point in the region for (*i*) $r < a$ and (*ii*) $r > a$.

[Delhi 2010]

Ans.

In these types of questions, first of all we have to calculate the current per unit area of cross-section so that we can calculate the current in each loop, then only we can find the magnetic field.

The current is distributed uniformly across the cross-section of radius a.



$\therefore$ Current passes per unit cross-section = $\dfrac{I}{\pi a^2}$

$\therefore$ Current passes through the cross-section of radius r is

$$I' = \left(\frac{I}{\pi a^2} \times \pi r^2\right) = \frac{Ir^2}{a^2} \quad \text{...(i) (1/2)}$$

(i) Consider a loop of radius r whose centre lies at the axis of wire where, $r < a$ as shown in figure inside the wire.

Applying Ampere's circuital law,

$$\oint \mathbf{B} \cdot dl = \mu_0 I' \quad \textbf{(1/2)}$$

$$\oint Bdl \cos 0° = \mu_0 \left(\frac{Ir^2}{a^2}\right) \quad \text{[From Eq. (i)]}$$

$$B \oint dl = \mu_0 \frac{Ir^2}{a^2}$$

$$B \times 2\pi r = \frac{\mu_0 Ir^2}{a^2} \Rightarrow B = \frac{\mu_0 Ir}{2\pi a^2} \quad \textbf{(1/2)}$$

$$\Rightarrow \quad B \propto r$$

(ii) Considering a loop of radius r whose centre lies at the axis of wire and $(r > a)$ as shown in outer dotted line.

$\therefore$ Current I threads the loops.

Applying Ampere's circuital law,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I \quad \textbf{(1/2)}$$

$$\oint Bdl \cos 0° = \mu_0 I$$

$$B \oint dl = \mu_0 I$$

$$B \times 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r} \quad \textbf{(1)}$$

$$\Rightarrow \quad B \propto \frac{1}{r}$$

22.A solenoid of length 1.0 m has a radius of 1 cm and has a total of 1000 turns wound on it. It carries a current of 5 A. Calculate the magnitude of the axial magnetic field inside the solenoid. If an electron was to move with a speed of 10^4 m/s along the axis of this current carrying solenoid, what would be the force experienced by this electron? [Delhi 2008 C]

Ans.

- We have to calculate the axial magnetic field inside the solenoid. In a solenoid, the magnetic field is along its axis, so it is called axial magnetic field. So, to find the axial magnetic field inside the solenoid, its regular formula will be used.

Given, $L = 1\text{m}$, $r = 1\text{cm} = 0.01$ m,

$N = 1000$, $I = 5$ A

∴ Magnetic field P inside the solenoid

$$B = \mu_0 nI \quad \textbf{(1/2)}$$

$$= \mu_0 \left(\frac{N}{L}\right) I$$

$$= \mu_0 \left(\frac{1000}{1}\right) \times 5$$

$$= 4\pi \times 10^{-7} \times 1000 \times 5$$

$$B = 2\pi \times 10^{-3}\ \text{T} \quad \textbf{(1½)}$$

The direction of B is along the axis of solenoid.

Now, $q = -e$,

$v = 10^4$ m/s

and the angle between B and v is 0°

(∵ electron moves along the direction of the magnetic field)

∴ Magnetic Lorentz force,

$$F_B = qvB \sin 0°$$

$$= qvB \times 0 = 0$$

$$F_B = 0$$

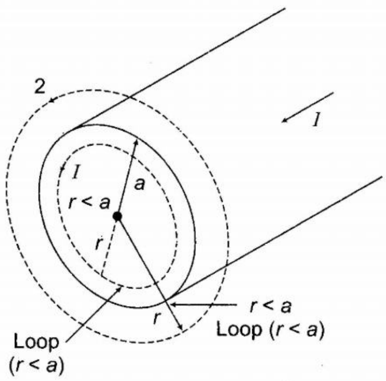
⇒ No magnetic force experienced by the electron. **(1)**

23.A long straight wire of a circular cross-section of radius a carries a steady current /. The current is uniformly distributed across the cross-section of the wire. Use Ampere’s circuital law to show that the magnetic field due to this wire in the region inside the wire increases in direct proportion to the distance of the field point from the axis of the wire. Write the value of this magnetic field on the surface of the wire.[All India 2008 C]

Ans.

In these types of questions, first of all we have to calculate the current per unit area of cross-section so that we can calculate the current in each loop, then only we can find the magnetic field.

The current is distributed uniformly across the cross-section of radius a.



$\therefore$ Current passes per unit cross-section $= \dfrac{I}{\pi a^2}$

$\therefore$ Current passes through the cross-section of radius r is

$$I' = \left(\frac{I}{\pi a^2} \times \pi r^2\right) = \frac{Ir^2}{a^2} \quad \text{...(i)} \quad \mathbf{(1/2)}$$

(i) Consider a loop of radius r whose centre lies at the axis of wire where, $r < a$ as shown in figure inside the wire.

Applying Ampere's circuital law,

$$\oint \mathbf{B} \cdot dl = \mu_0 I' \quad \mathbf{(1/2)}$$

$$\oint Bdl \cos 0° = \mu_0 \left(\frac{Ir^2}{a^2}\right) \quad \text{[From Eq. (i)]}$$

$$B \oint dl = \mu_0 \frac{Ir^2}{a^2}$$

$$B \times 2\pi r = \frac{\mu_0 Ir^2}{a^2} \Rightarrow B = \frac{\mu_0 Ir}{2\pi a^2} \quad \mathbf{(1/2)}$$

$$\Rightarrow \quad B \propto r$$

$$\because \quad B = \frac{\mu_0 I}{2\pi a^2} r$$

$$\Rightarrow \quad B \propto r \quad \mathbf{(1/2)}$$

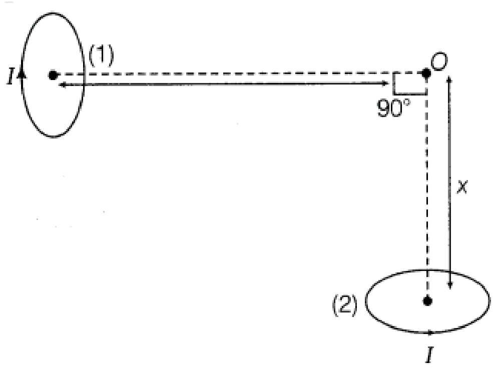
Now, the value of magnetic field on the surface of wire, i.e.

$$r = a$$

$$B = \frac{\mu_0 I}{2\pi a^2} \times a = \frac{\mu_0 I}{2\pi a} \quad \mathbf{(1/2)}$$

5 Marks Questions

24.Two very small identical circular loop(1) and (2) carrying equal current I are placed vertically (with respect to the plane of the paper) with their geometrical axes perpendicular to each other as shown in the figure. Find the magnitude and direction of the net magnetic field produced at the point O.[Delhi 2014]



Ans.

The magnetic field at a point due to a circular loop is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{(a^2 + r^2)^{3/2}}$$ **(1)**

where,

I = current through the loop

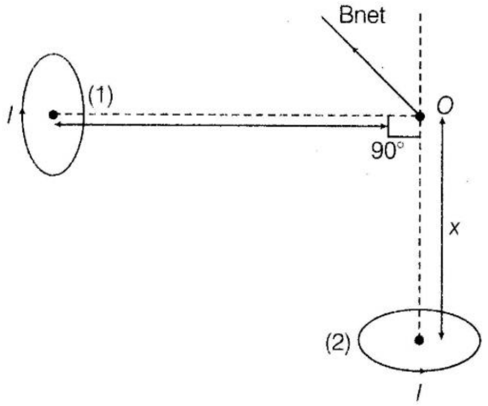
a = radius of the loop

r = distance of O from the centre of the loop.

Since I, a and $r = x$ are the same for both the loops, the magnitude of B will be the same and is given by **(1)**

$$B_1 = B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{(a^2 + x^2)^{3/2}}$$

The direction of magnetic field due to loop (1) will be away from O and that of the magnetic field due to loop (2) will be towards O as shown. The direction of the net magnetic field will be as shown below: **(1)**



(1)

The magnitude of the net magnetic field is given by

$$B_{net} = \sqrt{B_1^2 + B_2^2}$$

$$\Rightarrow \quad B_{net} = \frac{\mu_0}{4\pi} \frac{2\sqrt{2}\pi I a^2}{(a^2 + x^2)^{3/2}}$$ **(1)**

25. State Biot-Savart's law expressing it in the vector form. Use it to obtain the expression for the magnetic field at an axial point distance d from the centre of a circular coil of radius a carrying current I. Also, find the ratio of the magnitudes of the magnetic field of this coil at the centre and at an axial point for which $d = a\sqrt{3}$.

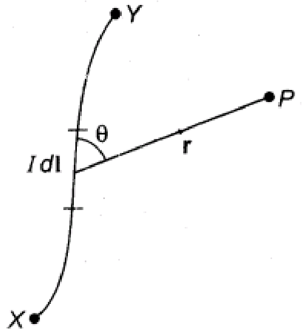
[Delhi 2013C]

Ans..

Biot-Savart's law This law states that the magnetic field (dB) at point P due to small current element Idl of current carrying conductor is

(*i*) directly proportional to the Idl (current) element of the conductor.

$dB \propto Idl$



(*ii*) directly proportional to $\sin\theta$

$dB \propto \sin\theta$

where, θ is the angle between dl and $\mathbf{r}$.

(*iii*) inversely proportional to the square of the distance of point P from the current element.

$$dB \propto \frac{1}{r^2}$$

(1)

Combining all the inequalities

$$dB \propto \frac{Idl \sin\theta}{r^2}$$

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin\theta}{r^2}$$

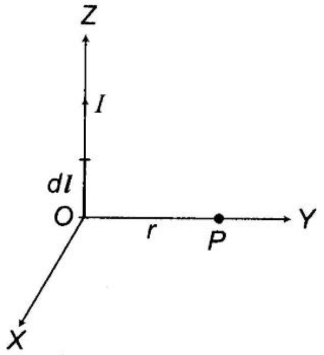
where, $\frac{\mu_0}{4\pi} = 10^{-7}$ T-m/A for free space.

The direction of magnetic field can be obtained using right hand thumb rule.

In vector form,

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{Id\mathbf{l} \times \hat{\mathbf{r}}}{|\mathbf{r}|^2}$$

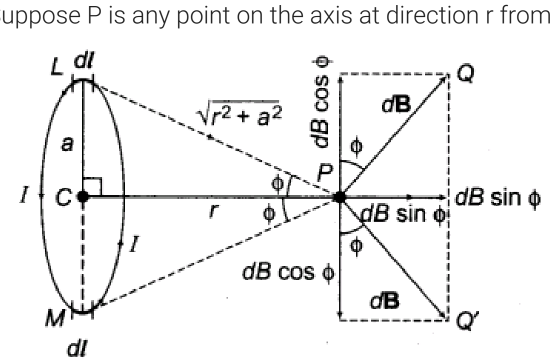
(1/2)



(1/2)

The direction of magnetic field will be perpendicular to Y-axis along upward in the plane of paper.

(i)Let us consider a circular loop of radius a with centre C. Let the plane of the coil be perpendicular to the plane of the paper and current / be flowing in the direction shown. Suppose P is any point on the axis at direction r from the centre.



Now, consider a current element Idl on top L, where current comes out of paper normally, whereas at bottom M enters into the plane paper normally

$\because \qquad LP \perp dl$

Also, $\qquad MP \perp dl$

$$LP = MP = \sqrt{r^2 + a^2}$$

The magnetic field at P due to current element Idl. According to Biot-Savart's law,

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin 90°}{(r^2 + a^2)}$$

where, $a =$ radius of circular loop.
$r =$ distance of point P from centre along the axis. **(1)**

The direction of dB is perpendicular to LP and along PQ, where $PQ \perp LP$. Similarly, the same magnitude of magnetic field is obtained due to current element Idl at the bottom and direction is along PQ', where $PQ' \perp MP$. Now, resolving dB due to current element at L and $MdB \cos\phi$ components balance each other and net magnetic field is given by

$$B = \oint dB \sin\phi$$

$$= \oint \frac{\mu_0}{4\pi}\left(\frac{Idl}{r^2 + a^2}\right) \cdot \frac{a}{\sqrt{r^2 + a^2}}$$

$$\left[\because \text{In } \Delta PCL, \sin\phi = \frac{a}{\sqrt{r^2 + a^2}}\right]$$

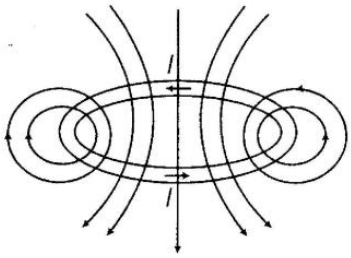
$$B = \frac{\mu_0}{4\pi} \frac{Ia}{(r^2 + a^2)^{3/2}} \oint dl$$

$$B = \frac{\mu_0}{4\pi} \frac{Ia}{(r^2 + a^2)^{3/2}} (2\pi a) = \frac{\mu_0 Ia^2}{2(r^2 + a^2)^{3/2}}$$

For n turns, $\qquad B = \dfrac{\mu_0 n Ia^2}{2\,(r^2 + a^2)^{3/2}}$ **(1)**

The direction is along the axis and away from the loop.

(*ii*) Magnetic field lines due to a current-carrying loop is given as below: **(2)**



In this answer, put $r = d$.
Magnetic field induction at the centre of the circular coil carrying current is

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I}{a}$$

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi a^2 I}{(a^2 + d^2)^{3/2}}$$

$$\frac{B_1}{B_2} = \frac{a^2 \times a}{(a^2 + d^2)^{3/2}} = \frac{a^3}{(a^2 + d^2)^{3/2}}$$

$[\because d = a\sqrt{3}]$

$$\frac{B_1}{B_2} = \frac{a^3}{(a^2 + 3a^2)^{3/2}} = \frac{a^3}{(4a^2)^{3/2}}$$

$$\frac{B_1}{B_2} = \frac{1}{8}$$

(3)

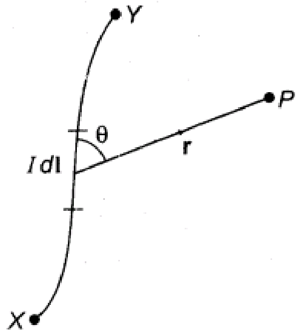
26.State Biot-Savart's law and give the mathematical expression for it. Use law to derive the expression for the magnetic field due to a circular coil carrying current at a point along its axis. How does a circular loop carrying current behave as a magnet?[Delhi 2011]

Ans.For Biot-Savart's law

Biot-Savart's law This law states that the magnetic field (dB) at point P due to small current element Idl of current carrying conductor is

(i) directly proportional to the Idl (current) element of the conductor.

$dB \propto Idl$



(*ii*) directly proportional to $\sin\theta$

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where, θ is the angle between dl and $\mathbf{r}$.

(*iii*) inversely proportional to the square of the distance of point P from the current element.

$$dB \propto \frac{1}{r^2}$$ **(1)**

Combining all the inequalities

$$dB \propto \frac{Idl\sin\theta}{r^2}$$

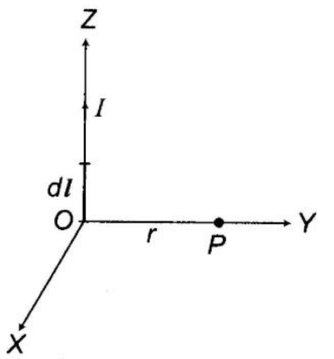
$$dB = \frac{\mu_0}{4\pi}\cdot\frac{Idl\sin\theta}{r^2}$$

where, $\frac{\mu_0}{4\pi} = 10^{-7}$ T-m/A for free space.

The direction of magnetic field can be obtained using right hand thumb rule.

In vector form,

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\cdot\frac{Id\mathbf{l}\times\hat{\mathbf{r}}}{|\mathbf{r}|^2}$$ **(1/2)**

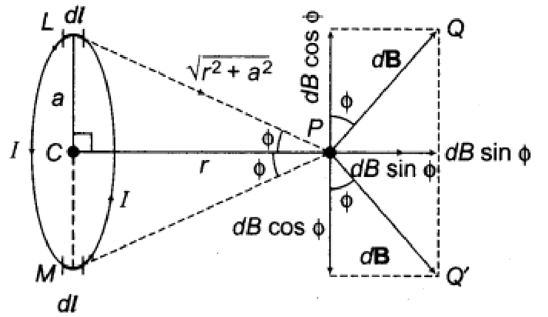


(1/2)

The direction of magnetic field will be perpendicular to Y-axis along upward in the plane of paper.

For the magnetic field due to a circular coil carrying current at a point along its axis

(i) Let us consider a circular loop of radius a with centre C. Let the plane of the coil be perpendicular to the plane of the paper and current / be flowing in the direction shown. Suppose P is any point on the axis at direction r from the centre.



Now, consider a current element **Idl** on top L, where current comes out of paper normally, whereas at bottom **M** enters into the plane paper normally

$\because \quad LP \perp dl$

Also, $\quad MP \perp dl$

$$LP = MP = \sqrt{r^2 + a^2}$$

The magnetic field at P due to current element Idl. According to Biot-Savart's law,

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin 90°}{(r^2 + a^2)}$$

where, $a =$ radius of circular loop.

$r =$ distance of point P from centre along the axis. **(1)**

The direction of dB is perpendicular to LP and along PQ, where $PQ \perp LP$. Similarly, the same magnitude of magnetic field is obtained due to current element Idl at the bottom and direction is along PQ', where $PQ' \perp MP$. Now, resolving dB due to current element at L and $MdB \cos \phi$ components balance each other and net magnetic field is given by

$$B = \oint dB \sin \phi$$

$$= \oint \frac{\mu_0}{4\pi} \left(\frac{Idl}{r^2 + a^2} \right) \cdot \frac{a}{\sqrt{r^2 + a^2}}$$

$$\left[\because \text{In } \Delta PCL, \sin \phi = \frac{a}{\sqrt{r^2 + a^2}} \right]$$

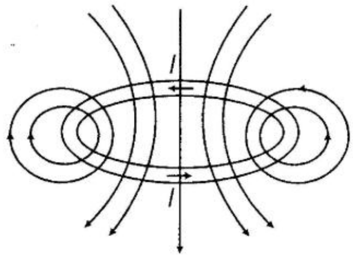
$$B = \frac{\mu_0}{4\pi} \frac{Ia}{(r^2 + a^2)^{3/2}} \oint dl$$

$$B = \frac{\mu_0}{4\pi} \frac{Ia}{(r^2 + a^2)^{3/2}} (2\pi a) = \frac{\mu_0 I a^2}{2(r^2 + a^2)^{3/2}}$$

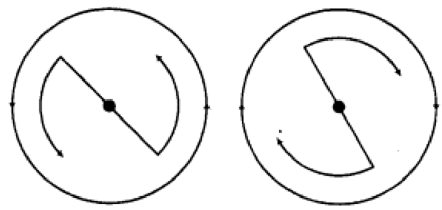
For n turns, $\quad B = \dfrac{\mu_0 n I a^2}{2\,(r^2 + a^2)^{3/2}}$ **(1)**

The direction is along the axis and away from the loop.

(*ii*) Magnetic field lines due to a current-carrying loop is given as below: **(2)**



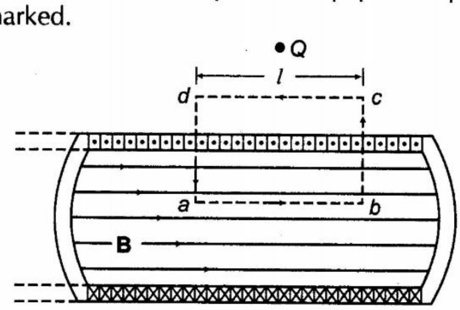
As current carrying loop has the magnetic held lines around it which exists a force on a moving charge. Thus, it behaves as a magnet with two mutually opposite poles



The anti-clockwise flow of current behaves like a North pole, whereas clockwise flow as South pole. Hence, loop behaves as a magnet

27.(i) Using Ampere's circuital law, obtain the expression for the magnetic field due to a long solenoid at a point inside the solenoid on its axis.
(ii) In what respect, is a toroid different from a solenoid? Draw and compare the pattern of the magnetic field lines in the two cases.
(iii) How is the magnetic field inside a given solenoid made strong?[All India 2011]
Ans.(i)

Figure shows the longitudinal sectional view of long current carrying solenoid. The current comes out of the plane of paper at points marked.



The **B** is the magnetic field at any point inside the solenoid.

Considering the rectangular closed path *abcda*. Applying Ampere's circuital law over loop *abcda*. **(1)**

$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times$ (Total current passing through loop *abcd*)

$$\int_a^b \mathbf{B} \cdot d\mathbf{l} + \int_b^c \mathbf{B} \cdot d\mathbf{l} + \int_c^d \mathbf{B} \cdot d\mathbf{l} + \int_d^a \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(\frac{N}{L} li\right)$$

where, $\frac{N}{L}$ = number of turns per unit length

$ab = cd = l$ = length of rectangle.

$$\int_a^b Bdl \cos 0° + \int_b^c Bdl \cos 90° + 0 + \int_d^a Bdl \cos 90° = \mu_0 \left(\frac{N}{L}\right) li$$

$$B \int_a^b dl = \mu_0 \left(\frac{N}{L}\right) li \Rightarrow Bl = \mu_0 \left(\frac{N}{L}\right) li$$

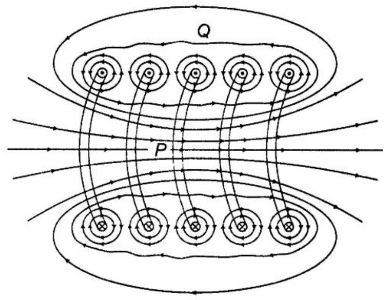
$$\Rightarrow \quad B = \mu_0 \left(\frac{N}{L}\right) i \quad \text{or} \quad B = \mu_0 ni$$ **(1)**

where, n = number of turns per unit length. This is required expression for magnetic field inside the long current carrying solenoid.

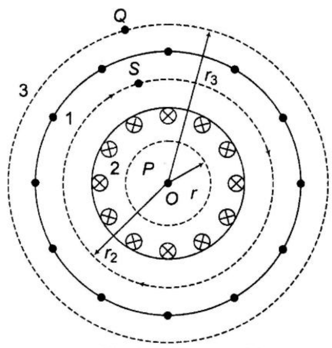
(ii) Solenoid is a hollow circular ring having large number of turns of insulated copper wire on it.

Therefore, we can assume that toroid is a bent solenoid to close on itself.

The magnetic fields due to solenoid and toroid is given in figures below



Field due to solenoid



Field due to toroid

Magnetic field inside the solenoid is a uniform, strong and along its axis also field lines are all most parallel while inside the toroid field line makes closed path.

(iii) The magnetic field in the solenoid can be increased by inserting a soft iron core inside it.

28.(i) State Ampere's circuital law.

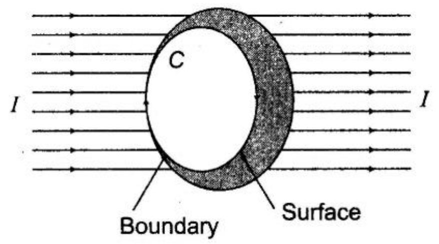
(ii) Use it to derive an expression for magnetic field inside along the axis of an air cored solenoid.

(iii) Sketch the magnetic field lines for a finite solenoid. How are these field lines different from the electric field lines from an electric dipole? [Foreign 2010]

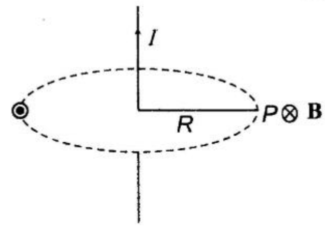
Ans.(i) For statement of Ampere's circuital law

As, Ampere's circuital law states that the line integral of magnetic field **B** around any closed loop is equal to μ_0 times the total current threading through the loop. **(1)**

i.e. $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$



To explain the Ampere's circuital law consider an infinitely long conductor wire carrying a steady current I as shown in the figure.



In order to determine the magnetic field at point P which is situated at a distance R from the centre of the circular loop around the conductor wire **B** (magnetic field) is tangential to circumference of the loop. **(1)**

Now, $\oint \mathbf{B} \cdot d\mathbf{l} = \int B\,dl = B\,2\pi R$

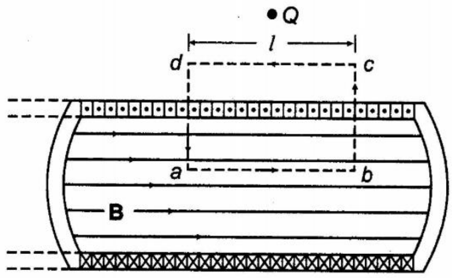
$$= \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi R}$$

[From Ampere's circuital law]

The direction of magnetic field will be determined by right hand rule.

(ii)

Figure shows the longitudinal sectional view of long current carrying solenoid. The current comes out of the plane of paper at points marked.



The **B** is the magnetic field at any point inside the solenoid.

Considering the rectangular closed path *abcda*. Applying Ampere's circuital law over loop *abcda*. **(1)**

$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times$ (Total current passing through loop *abcd*)

$$\int_a^b \mathbf{B} \cdot d\mathbf{l} + \int_b^c \mathbf{B} \cdot d\mathbf{l} + \int_c^d \mathbf{B} \cdot d\mathbf{l} + \int_d^a \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(\frac{N}{L} li \right)$$

where, $\frac{N}{L}$ = number of turns per unit length

$ab = cd = l$ = length of rectangle.

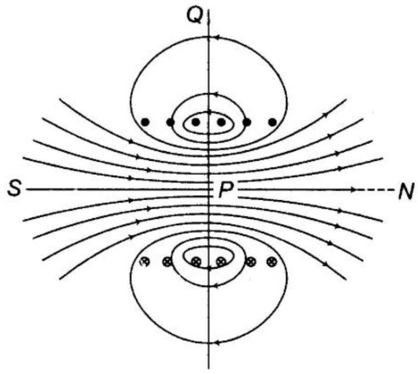
$$\int_a^b Bdl \cos 0° + \int_b^c Bdl \cos 90° + 0 + \int_d^a Bdl \cos 90° = \mu_0 \left(\frac{N}{L} \right) li$$

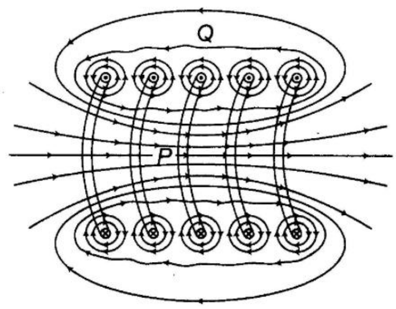
$$B \int_a^b dl = \mu_0 \left(\frac{N}{L} \right) li \Rightarrow Bl = \mu_0 \left(\frac{N}{L} \right) li$$

$$\Rightarrow \qquad B = \mu_0 \left(\frac{N}{L} \right) i \quad \text{or} \quad B = \mu_0 ni$$ **(1)**

where, n = number of turns per unit length. This is required expression for magnetic field inside the long current carrying solenoid.

(*iii*) Magnetic field lines due to a finite solenoid has been shown as below:





(2)

All the magnetic field lines are necessarily closed loops, whereas electric lines of force are not.

29.(i)Using Biot-Savart's law, deduce an expression for the magnetic field on the axis of a circular current carrying loop.

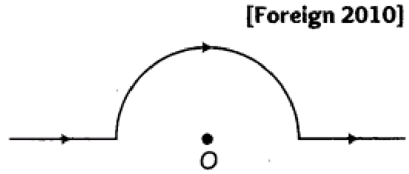
(ii) Draw the magnetic field lines due to a current carrying loop.

(iii) A straight wire carrying a current of 12 A is bent into a semi-circular arc of radius 2.0 cm as shown in the figure. What is the magnetic field B at O due to

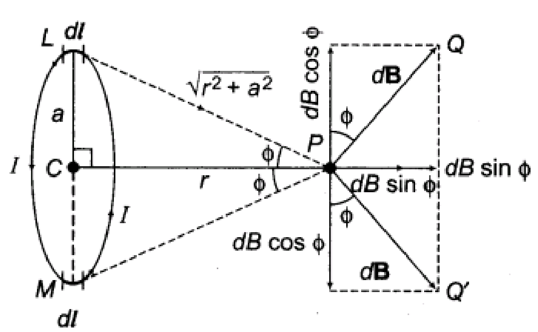
(a)straight segments,

(b)the semi-circular arc?[Foreign 2010]

[Foreign 2010]



Ans.(i) Let us consider a circular loop of radius a with centre C. Let the plane of the coil be perpendicular to the plane of the paper and current / be flowing in the direction shown. Suppose P is any point on the axis at direction r from the centre.



Now, consider a current element **Idl** on top L, where current comes out of paper normally, whereas at bottom **M** enters into the plane paper normally

$\because$ $LP \perp dl$

Also, $MP \perp dl$

$$LP = MP = \sqrt{r^2 + a^2}$$

The magnetic field at P due to current element Idl. According to Biot-Savart's law,

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin 90^\circ}{(r^2 + a^2)}$$

where, $a =$ radius of circular loop.
$r =$ distance of point P from centre along the axis. **(1)**

The direction of dB is perpendicular to LP and along PQ, where $PQ \perp LP$. Similarly, the same magnitude of magnetic field is obtained due to current element Idl at the bottom and direction is along PQ', where $PQ' \perp MP$. Now, resolving dB due to current element at L and $MdB \cos \phi$ components balance each other and net magnetic field is given by

$$B = \oint dB \sin \phi$$

$$= \oint \frac{\mu_0}{4\pi} \left(\frac{Idl}{r^2 + a^2} \right) \cdot \frac{a}{\sqrt{r^2 + a^2}}$$

$$\left[\because \text{In } \Delta PCL, \sin \phi = \frac{a}{\sqrt{r^2 + a^2}} \right]$$

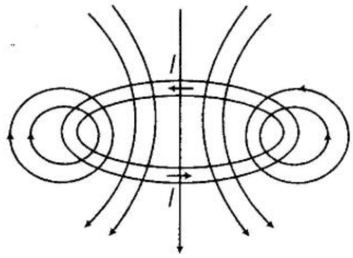
$$B = \frac{\mu_0}{4\pi} \frac{Ia}{(r^2 + a^2)^{3/2}} \oint dl$$

$$B = \frac{\mu_0}{4\pi} \frac{Ia}{(r^2 + a^2)^{3/2}} (2\pi a) = \frac{\mu_0 Ia^2}{2(r^2 + a^2)^{3/2}}$$

For n turns, $$B = \frac{\mu_0 n Ia^2}{2 (r^2 + a^2)^{3/2}}$$ **(1)**

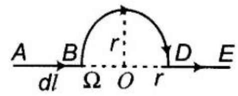
The direction is along the axis and away from the loop.

(*ii*) Magnetic field lines due to a current-carrying loop is given as below: **(2)**



(*iii*) Magnetic field due to straight part

$$B = \int \frac{\mu_0}{4\pi} \frac{I d\mathbf{l} \times \mathbf{r}}{r^3}$$



(1/2)

For point O, $d\mathbf{l}$ and r for each element of the straight segments AB and DE are parallel. Therefore, $d\mathbf{l} \times \mathbf{r} = 0$. Hence, magnetic field due to straight segments is zero.

Magnetic field at the centre due to circular point

$$= \frac{\text{Magnetic field at the centre of circular coil}}{2}$$

[∵ Here, coil is half]

$$= \frac{1}{2}\left(\frac{\mu_0 I}{2r}\right) = \frac{\mu_0 I}{4r}$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4r}$$

$$= \frac{(4\pi \times 10^{-7}) \times 12}{4 \times 2 \times 10^{-2}}$$

$$= 6\pi \times 10^{-5} \text{ T}$$ **(1/2)**

30.(i) State Ampere's circuital law. Show through an example, how this law enables an easy evaluation of this magnetic field when there is a symmetry in the system?

(ii) What does a toroid consist of? Show that for an ideal toroid of closely wound turns, the magnetic field.

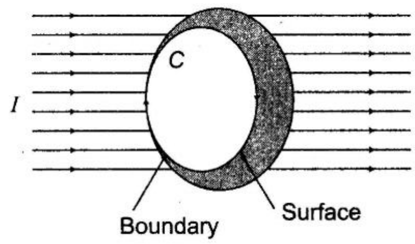
(a)inside the toroid is constant.

(b)in the open space inside an exterior to the toroid is zero.[All India 2010 C]

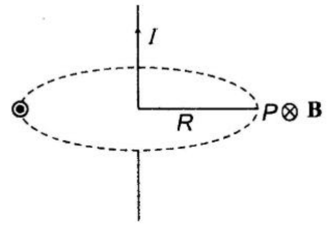
Ans.(i) For statement of Ampere's law

As, Ampere's circuital law states that the line integral of magnetic field **B** around any closed loop is equal to μ_0 times the total current threading through the loop. **(1)**

i.e. $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$



To explain the Ampere's circuital law consider an infinitely long conductor wire carrying a steady current I as shown in the figure.



In order to determine the magnetic field at point P which is situated at a distance R from the centre of the circular loop around the conductor wire **B** (magnetic field) is tangential to circumference of the loop. **(1)**

Now, $\oint \mathbf{B} \cdot d\mathbf{l} = \int B\, dl = B\, 2\pi R$

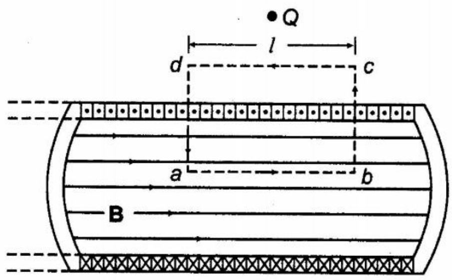
$$= \mu_0 I \Rightarrow B = \frac{\mu_0 I}{2\pi R}$$

[From Ampere's circuital law]

The direction of magnetic field will be determined by right hand rule.

For the evaluation of magnetic field for a symmetrical system, we can consider the example of a current carrying solenoid. Now,

Figure shows the longitudinal sectional view of long current carrying solenoid. The current comes out of the plane of paper at points marked.



The **B** is the magnetic field at any point inside the solenoid.

Considering the rectangular closed path *abcda*. Applying Ampere's circuital law over loop *abcda*. **(1)**

$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times$ (Total current passing through loop *abcd*)

$$\int_a^b \mathbf{B} \cdot d\mathbf{l} + \int_b^c \mathbf{B} \cdot d\mathbf{l} + \int_c^d \mathbf{B} \cdot d\mathbf{l} + \int_d^a \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(\frac{N}{L} li\right)$$

where, $\frac{N}{L}$ = number of turns per unit length

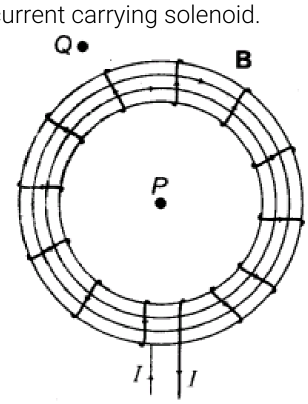
$ab = cd = l$ = length of rectangle.

$$\int_a^b Bdl \cos 0° + \int_b^c Bdl \cos 90° + 0 + \int_d^a Bdl \cos 90° = \mu_0 \left(\frac{N}{L}\right) li$$

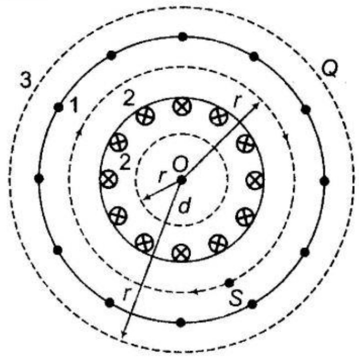
$$B \int_a^b dl = \mu_0 \left(\frac{N}{L}\right) li \Rightarrow Bl = \mu_0 \left(\frac{N}{L}\right) li$$

$$\Rightarrow \quad B = \mu_0 \left(\frac{N}{L}\right) i \quad \text{or} \quad B = \mu_0 ni$$ **(1)**

where, n = number of turns per unit length. This is required expression for magnetic field inside the long current carrying solenoid.



(a) Let magnetic field inside the toroid is B along the considered loop (1) as shown in figure.



Applying Ampere's circuital law,

$$\oint_{\text{loop 1}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 (NI)$$

Since, toroid of N turns, threads the loop 1, N times, each carrying current I inside the loop. Therefore, total current threading the loop 1 is NI.

$$\Rightarrow \oint_{\text{loop 1}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 NI$$

$$B \oint_{\text{loop}} dl = \mu_0 NI$$

$$B \times 2\pi r = \mu_0 NI \quad \text{or} \quad B = \frac{\mu_0 NI}{2\pi r}$$ **(1)**

(b) **Magnetic field inside the open space interior the toroid.** Let the loop (2) is shown in figure experience magnetic field B.

No current threads the loop 2 which lie in the open space inside the toroid.

$\therefore$ Ampere's circuital law

$$\oint_{\text{loop 2}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 (\theta) = 0 \Rightarrow B = 0$$ **(1)**

Magnetic field in the open space exterior of toroid Let us consider a coplanar loop (3) in the open space of exterior of toroid. Here, each turn of toroid threads the loop two times in opposite directions.

Therefore, net current threading the loop

$$= NI - NI = 0$$

$\therefore$ By Ampere's circuital law,

$$\oint_{\text{loop 3}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 (NI - NI) = 0 \Rightarrow B = 0$$

Thus, there is no magnetic field in the open space interior and exterior of toroid. **(1)**